Fermi National Accelerator Laboratory

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Calculation of Electron Polarization in High-Energy Storage Rings Including Transverse Momentum Recoils

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The Derbenev-Kondratenko polarization formula takes into account longitudinal recoils (energy loss) due to photon emissions by an electron. Various algorithms have been published to evaluate this formula for high-energy storage rings. One of these algorithms is extended to evaluate the extended Derbenev-Kondratenko formula which includes the effects of transverse momentum recoils, and the results are compared with those from the original Derbenev-Kondratenko formula.

I. INTRODUCTION

A formula for the equilibrium polarization of electrons in high-energy storage rings was published in 1973 by Derbenev and Kondratenko.[1] This formula takes into account longitudinal recoils (energy loss) by an electron caused by photon emissions. In addition, however, the photons also excite transverse momentum recoils, of relative magnitude mc^2/E, where m is the electron mass and E its energy. The effects of transverse momentum recoils are neglected in Ref. 1. Recently, polarization formulas have been published taking into account these transverse momentum recoils.[2,3,4] It was found in Ref. 2 that, in a perfectly aligned horizontal weak-focusing accelerator, vertical momentum recoils could excite a nontrivial spin resonance, which were not excited by the longitudinal recoils. In this paper, the polarization will be evaluated for various accelerator models, with and without transverse momentum recoils.

II. POLARIZATION FORMULAS

The Derbenev-Kondratenko polarization formula is[1]

$$P_{eq} = \frac{8}{5\sqrt{3}} \frac{\left\langle \frac{1}{|\rho|^3} \hat{b}.\left(\hat{n} - \gamma \frac{\partial \hat{n}}{\partial \gamma}\right) \right\rangle}{\left\langle \frac{1}{|\rho|^3} \left(1 - \frac{2}{9}(\hat{n}.\hat{v}\,)^2 + \frac{11}{18}\left|\gamma \frac{\partial \hat{n}}{\partial \gamma}\right|^2\right) \right\rangle}\,, \tag{1}$$

where ρ is the local radius of curvature of the particle trajectory, $\hat{v}$ is the direction of particle motion, $\hat{b} = \vec{v} \times \dot{\vec{v}}/|\vec{v} \times \dot{\vec{v}}|$, $\hat{n}$ is the spin quantization axis on the particle trajectory, and the angular brackets denote an ensemble average over the particle trajectories and accelerator azimuth (or arc-length). The derivative $\gamma(\partial\hat{n}/\partial\gamma)$ describes the effects of longitudinal electron recoils.

To generalize the above formula, it is convenient to use a right-handed set of unit vectors $\{\hat{\imath}, \hat{\jmath}, \hat{k}\}$, with $\hat{\imath}$ along the outward normal to the trajectory, $\hat{\jmath}$ along the tangent, and $\hat{k} = \hat{b}$, the direction of the local magnetic field. The electron spin quantization axis is called $\hat{n}$ (see Ref. 3 for details). Then the change in $\hat{n}$ due to a photon emission is given by the derivatives

$$\vec{d} = \gamma \frac{\partial \hat{n}}{\partial \gamma} , \qquad \vec{e} = \frac{\partial \hat{n}}{\partial \beta_i} , \qquad \vec{f} = \frac{\partial \hat{n}}{\partial \beta_k} , \tag{2}$$

where $\vec{\beta} = \vec{v}/c$ and γ is the electron energy in units of rest energy. The vector $\vec{d}$ arises from the longitudinal recoil of an electron while the vectors $\vec{e}$ and $\vec{f}$ arise from transverse recoils along $\hat{i}$ and $\hat{k}$, respectively. Then

$$P = \frac{8}{5\sqrt{3}} \frac{\left\langle \frac{1}{|\rho|^3} \left\{ \hat{b}.\hat{n} - \hat{b}.\vec{d} + \frac{1}{3\gamma}\hat{v}.\vec{f} - \frac{1}{3\gamma}\hat{n}.(\vec{d} \times \vec{e}) \right\} \right\rangle}{\left\langle \frac{1}{|\rho|^3} \left\{ 1 - \frac{2}{9}(\hat{n}.\hat{v})^2 + \frac{11}{18}|\vec{d}|^2 + \frac{13}{90\gamma^2}(|\vec{e}|^2 + |\vec{f}|^2) + \frac{1}{9\gamma}\frac{\dot{\vec{v}}}{|\dot{\vec{v}}|}.(\hat{n} \times \vec{f}) \right\} \right\rangle} . \tag{3}$$

An algorithm to evaluate Eq. (1) for a storage ring was published in Ref. 5, and has been extended to include Eq. (3). Numerical results are given below.

III. NUMERICAL RESULTS

Fig. 1 shows the result of calculation of the polarization, to zeroth order, for a simple model accelerator. The resonance is a vertical betatron resonance $\nu = Q_y$, where ν is the spin tune and Q_y is the vertical betatron tune. On the horizontal axis we plot the value of $a\gamma = E(GeV)/.440652$, where $a = (g-2)/2$. The model is a ten-fold symmetric horizontal ring (a FODO lattice) with a circumference of 1000 m, one rf cavity and one vertical kicker. In Fig. 1 the accelerator was perfectly aligned. In such a case it is known that the longitudinal recoils do not excite any spin resonances, and Eq. (1) predicts $P = 8/(5\sqrt{3})$ always. It was found in Ref. 2 that vertical recoils, however, do excite a spin resonance in a perfectly aligned ring. The model treated was a perfectly aligned planar weak-focusing accelerator. It was found that, as the energy of the accelerator increases through the resonant value, the polarization drops, then increases to 99.2%, then falls again to $8/(5\sqrt{3}) \simeq 92.4\%$. We see from Fig. 1 that something similar happens in a perfectly aligned strong-focusing accelerator also. In this model the polarization first increases, to

98%, then decreases, as the energy is increased. Also, in a strong-focusing accelerator, more than one spin resonance is excited. The resonance $\nu = Q_y + 10$ is also excited, as shown in Fig. 2. Because of the ten-fold symmetry of the ring, only resonances of the form $\nu = Q_y + 10k$, $k = 0, 1, 2, \ldots$ are excited. In Fig. 2 the polarization increases by only a small amount, from 92.4% to 93%, before decreasing to a low value.

In both Figs. 1 and 2 the accelerator was perfectly aligned. It is known that, in a real accelerator, closed orbit distortions enable the longitudinal electron recoils to excite spin resonances. In Fig. 3 the same model is used, but the r.m.s. vertical closed orbit distortion is 0.32 mm. The polarization still increases, very slightly, below the energy of the resonance, by a fraction of a percent, but obviously this is a much smaller effect than in Fig. 1. In Fig. 4 the r.m.s. closed orbit distortion is 1.1 mm, and the longitudinal recoil terms clearly dominate the behavior of the polarization. In Fig. 5, the polarization of the same model has been calculated over a larger energy range. The resonances are labelled in the figure. The solid curve is the polarization with only longitudinal recoils (Eq. (1)), while the dashed curve includes transverse recoils (Eq. (3)). The r.m.s. closed orbit distortion is 1.1 mm, the same as in Fig. 4. There is a distinct difference between the curves, showing that the transverse recoils do contribute a non-zero effect. The polarization does not increase to 98%, but it is about 2% higher in some places, and about 2% lower in others. The transverse recoils contribute mainly near the vertical betatron resonance $\nu = Q_y$.

It can be shown that, like the vector $\vec{d}$, the vector $\vec{e}$ (which describes horizontal transverse recoils in the above model), also vanishes in a perfectly aligned horizontal accelerator. The effects in Figs. 1 and 2 are due to $\vec{f}$. No effects due to $\vec{e}$ were observed. It appears to be negligible.

The above results were calculated in a spin tune range $4.5 < a\gamma < 5$, except for Fig. 2. At higher energies, the effects of transverse momentum recoils will be even smaller because of the $1/\gamma = mc^2/E$ factors which multiply the vectors $\vec{e}$ and $\vec{f}$ in Eq. (3). To check this, the polarization was calculated in the range $35 < a\gamma < 35.5$ and the result is shown in Fig. 6. The format of the the graph is the same as in Fig. 5. The accelerator is a twenty-fold symmetric ring with a circumference of 2000 m, and a vertical r.m.s. closed orbit distortion of 0.5 mm. We see that the effect of transverse momentum recoils in this model is too small to be observed.

All of the above results were for planar rings (possibly with closed orbit distortions), so that the spin quantization axis was always nearly vertical. In Figs. 7 and 8 results are shown for an accelerator with a simple model spin rotator pair. The "spin rotator" model is a thin element of zero length, which does not affect the orbit, but rotates the spin by $\pm 90^\circ$ around the $\hat{x}$ axis. The accelerator is the same as that used in Figs. 1 – 5, with one pair of rotators, to rotate spins from vertical to longitudinal and back again in one of the accelerator straight sections. There are no closed orbit distortions. The polarization, to zeroth order, is plotted in Fig. 7, in the same energy range as Fig. 5. The solid curve again shows the result of using only longitudinal recoils, and the dashed curve shows the result of including transverse recoils. Because the "spin rotator" model used does not affect the orbit, there is no vertical dispersion in the ring, hence the longitudinal recoils do not excite a vertical betatron resonance. However, because of the rotation of the axis $\hat{n}$ by the rotators, there *is* coupling to the horizontal betatron and synchrotron spin resonances, as seen in the figure. This is in agreement with known behavior of the first-order spin-integrals. The transverse recoils, as in Figs. 1 and 2, do excite a vertical betatron spin resonance. In Fig. 8 the same curves are plotted in more detail around the vertical betatron resonance. It is again seen that the transverse recoil terms in the polarization formula increase the polarization on one side of the vertical betatron resonance, and decrease it on the other. The effect is about 2% in magnitude.

IV. CONCLUSION

It has been shown that transverse momentum recoils do excite spin resonances in a perfectly aligned planar strong-focusing accelerator, and the polarization does increase above 92.4% on one side of the resonance, in agreement with the findings in Ref. 2. The effect of longitudinal recoils on this result, as the closed orbit distortion is increased, has also been demonstrated. In addition, the relative effects of longitudinal and transverse recoils, for a ring with an idealized spin rotator pair, are also shown. It is found that the effects of the longitudinal recoils generally dominate those of the transverse recoils when the accelerator is not perfectly planar. The relative effect of transverse momentum recoils also decreases with increasing energy.

The situation is similar to that of the vertical emittance in horizontal storage rings. For a perfectly aligned planar ring, the longitudinal electron recoils do not excite any vertical emittance, whereas the vertical momentum recoils do. The vertical emittance is therefore determined by the vertical recoils in this case. However, for a real accelerator, with closed orbit distortions, the longitudinal recoils yield the dominant contribution to the vertical emittance.

ACKNOWLEDGEMENTS

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[5] S.R. Mane, Phys. Rev. A **36**, 120 (1987).

Fig. 1

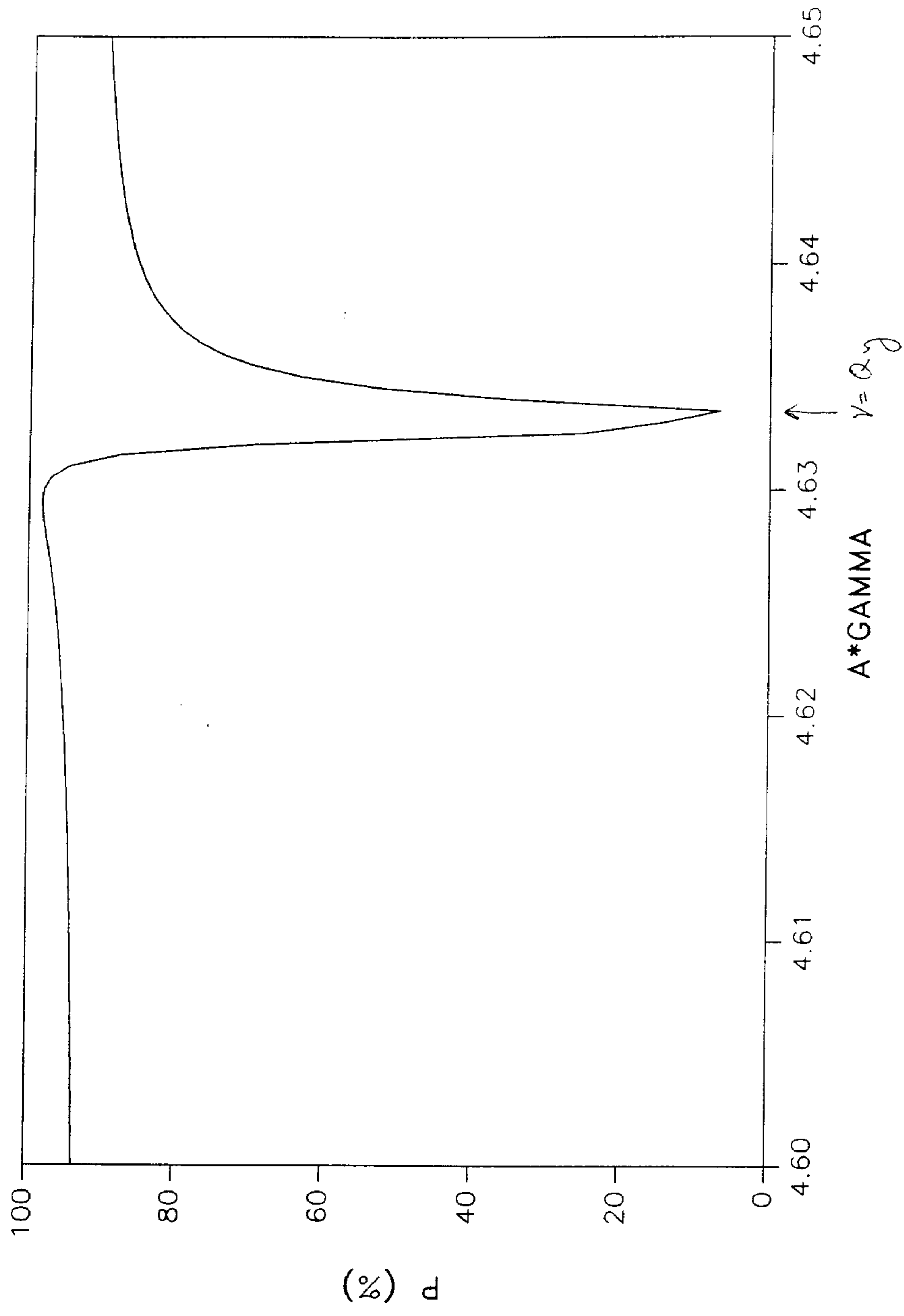


Fig. 2

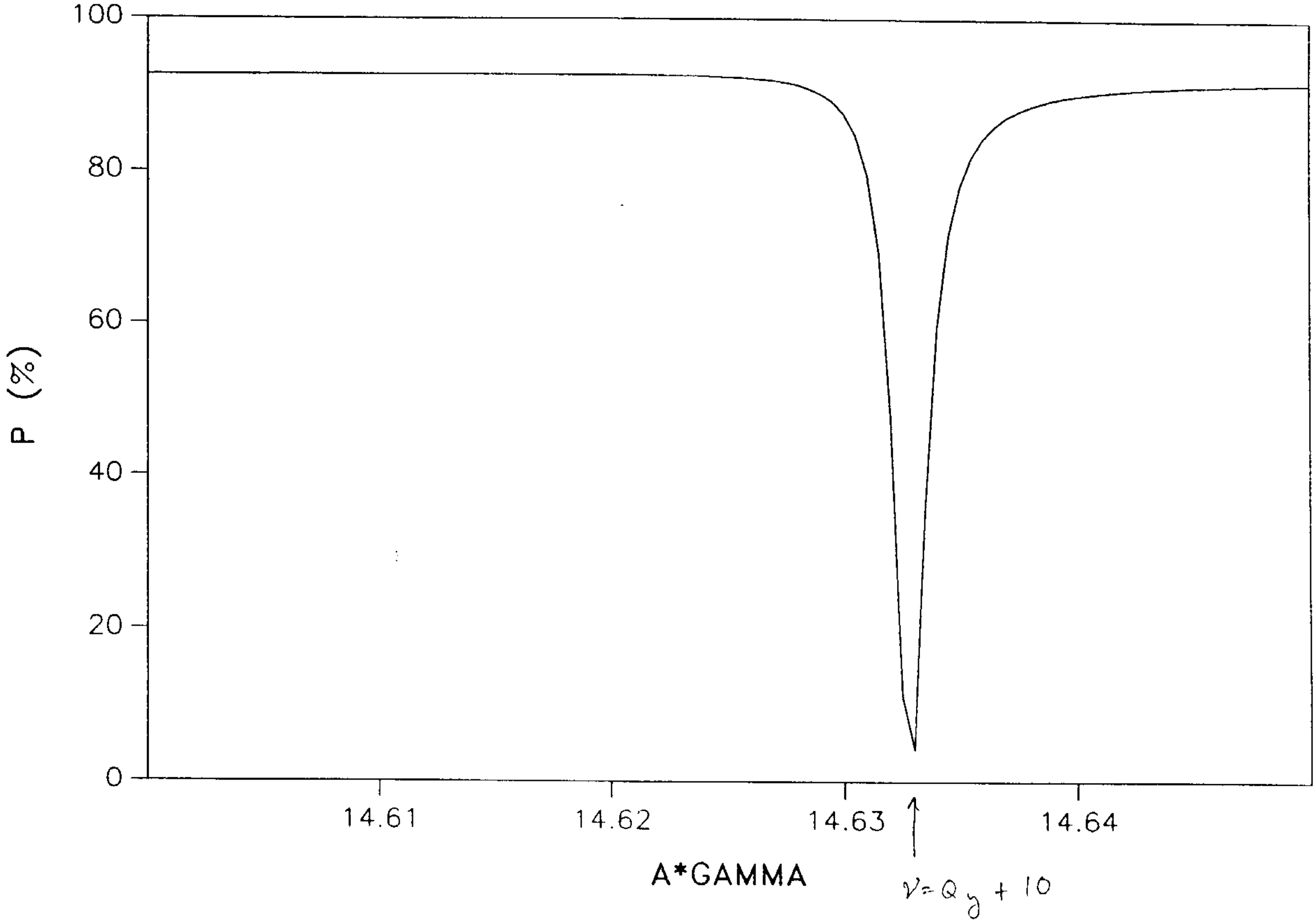


Fig. 3

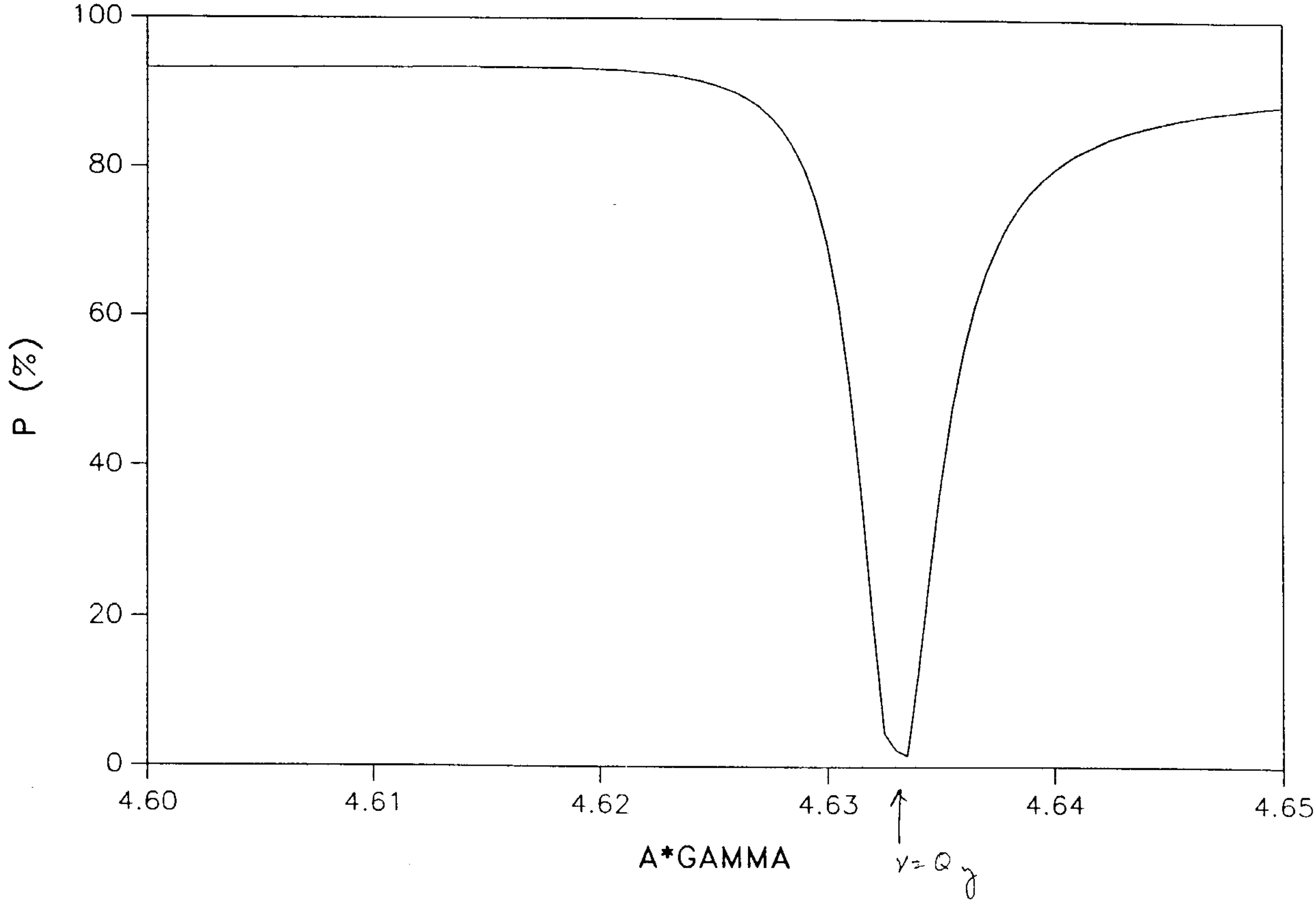
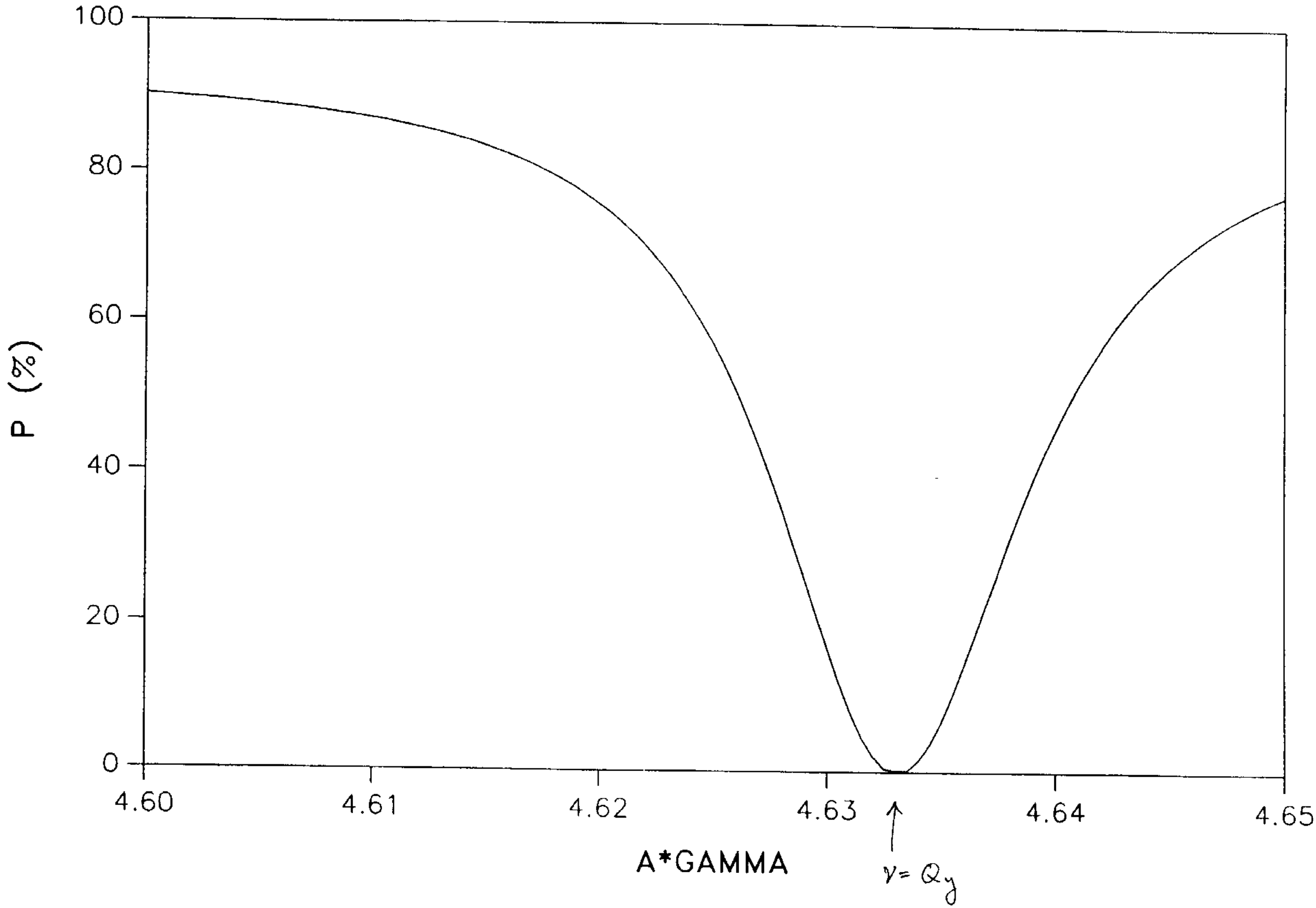


Fig 4



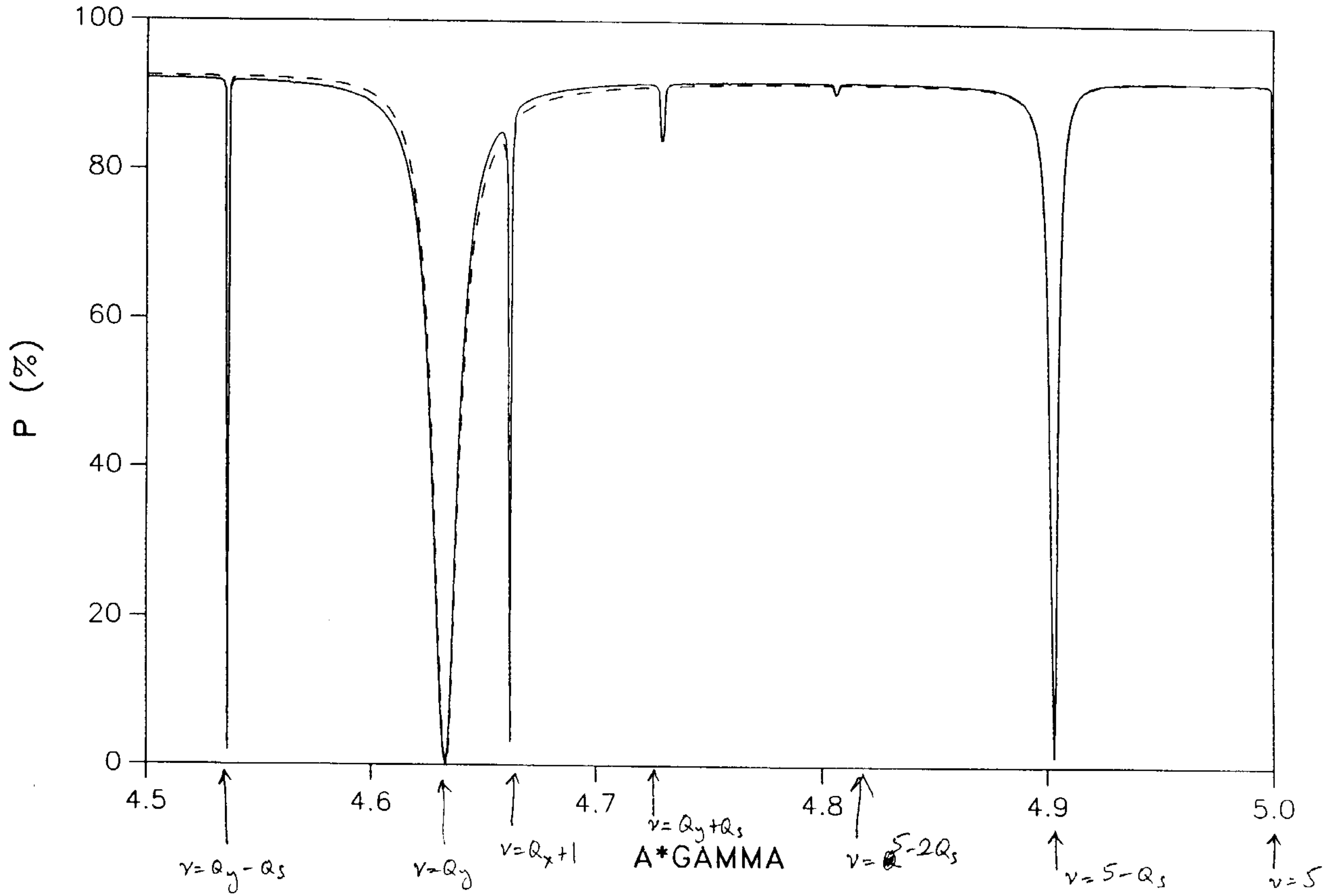
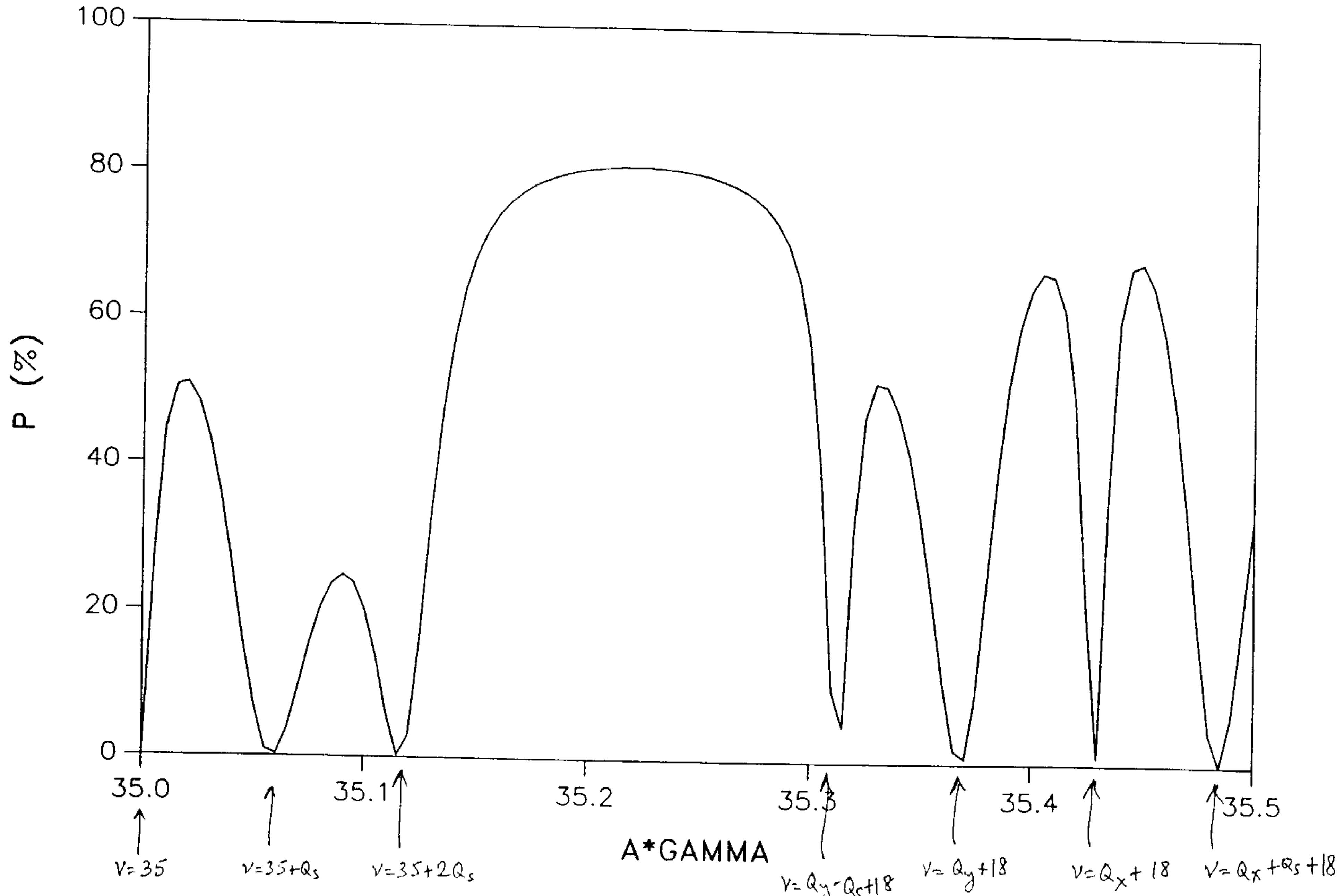

100
80
60
40
20
0
P (%)
4.5
4.6
4.7
4.8
4.9
5.0
ν = Qy - Qs
ν = Qy
ν = Qx + 1
ν = Qy + Qs
A*GAMMA
ν = 5 - 2Qs
ν = 5 - Qs
ν = 5

Fig 5

Fig. 6



P (%)
100
80
60
40
20
0
35.0
35.1
35.2
35.3
35.4
35.5
A*GAMMA
ν=35
ν=35+Qs
ν=35+2Qs
ν=Qy-Qs+18
ν=Qy+18
ν=Qx+18
ν=Qx+Qs+18

Fig. 7

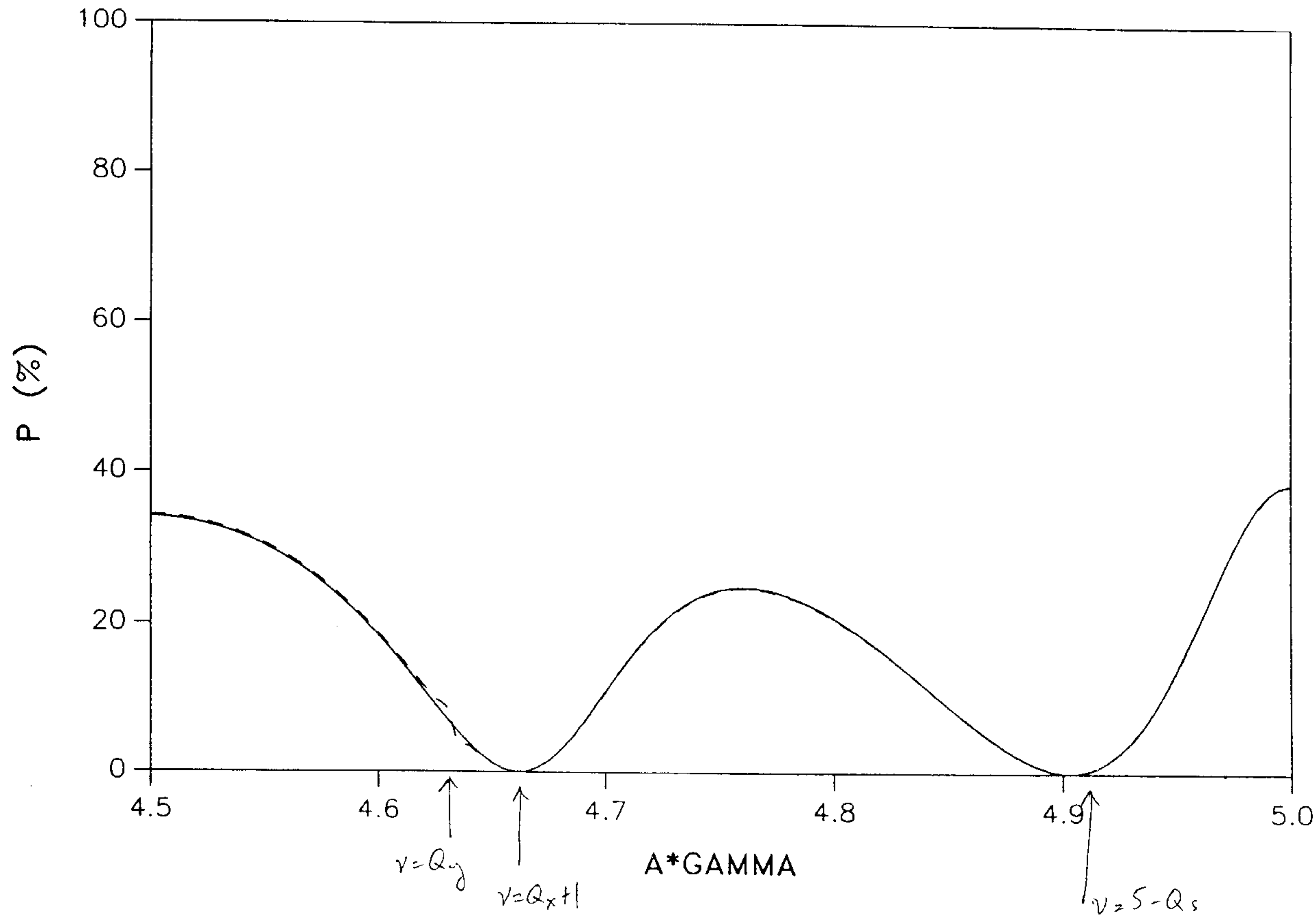
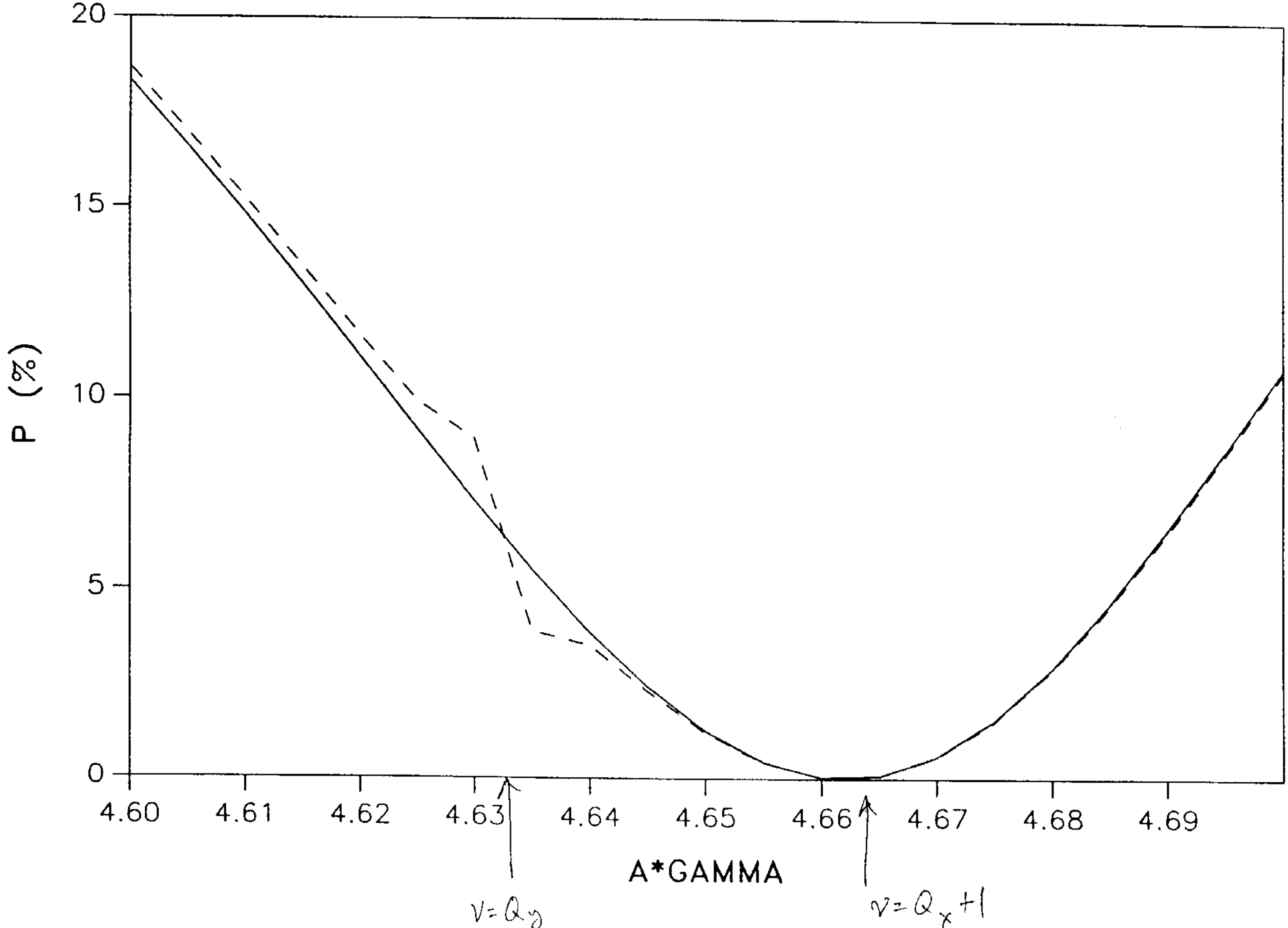


Fig. 8



20
15
10
5
0
P (%)
4.60
4.61
4.62
4.63
4.64
4.65
4.66
4.67
4.68
4.69
A*GAMMA
$\nu = Q_y$
$\nu = Q_x + 1$